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ON GEOSTROPHIC ADJUSTMENT PROCESS OF OCEANIC MOTIONS

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ABSTRACT

In this paper, some important features of the geostrophic adjustment process of large-scale motions in a barotropic and a baroclinic oceans are systematically investigated by means of the scale analysis. For the barotropic ocean, the constraint of the horizontal scale on the linear theory of geostrophic adjustment process is given and the dispersive mechanism of the unbalanced energy in the response of the flow field in the oceanic interior to a steady wind field is expounded. For the baroclinic ocean, the effects of the water depth and density stratification on the solution of geostrophic balance state are taken into account. As examples, some major properties of the geostrophic equilibrium state in the ocean of finite depth and in a two-layer model ocean which are different from those in the atmoshere are discussed in detail. In addition, a conservation equation for the nonlinear model is derived through introducing some simplications in which only the nonlinear potential flow and vertical transport of mass field are considered.

I. INTRODUCTION

The geostrophic adjustment process is one of the basic theoretical problems in the field of geophysical fluid dynamics. It was first posed and studied by Rossby (1936), later by Cahn, Bolin[1] and Veronis[2], who investigated the dispersive mechanisms of non-geostrophic energy in a barotropic or a baroclinic ocean as well as how the energy is partitioned to geostrophic and non-geostrophic motions. Other authors (for example, Obukhov, Ye Duzheng[3], Li Maicun, Zeng Qingcun[4], Chen Qiushi, Kuo, H. L.[5] and Bluman[6,7]) worked thoroughly at the details of the geostrophic adjustment process of atmospheric motions and systematically established the linear and non-linear geostrophic adjustment theories. As a result, the great success in the routing numerical weather prediction has been achieved with the applications of these theories.

By comparison, the development of the geostrophic adjustment theories of oceanic motions is not so rapid as that of atmospheric motions. The previous work by Rossby and Bolin was mainly concerned with the oceanic flow within a finite width. The conclusions thus obtained are restrictive to a certain degree. The thermodynamic process of the atmosphere is different from that of the ocean. Because the state equation of the ideal gas is valid for the atmosphere, the treatments of many problems related to the baroclinic atmosphere are simpler than those of the baroclinic oceanic ones. The limited oceanic bounds in the vertical and the density stratification exert directly influence on the properties of the solution of geostrophic equilibrium state, so many theoretical results obtained about the geostrophic adjustment of the baroclinic atmosphere are not adaptable to the baroclinic ocean. In this paper, based on the viewpoint of atmosphere-ocean dynamics, some important features of the geostrophic adjustment process in a barotrophic and a baroclinic ocean are systematically investigated by means of the scale analysis. For the barotropic ocean, the constraint of the horizontal scale on the linear theory of geostrophic adjustment process and the dispersive mechanism of the unbalanced energy in the response of the flow field in the oceanic interior to a steady wind field are revealed. For the baroclinic ocean, some main characteristics of the geostrophic equilibrium state in the ocean of finite depth and in a two-layer model ocean which differs from those in the atmosphere are discussed in detail. Finally, a potential vorticity conservation equation for the nonlinear adjustment process under some simplications is derived.

II. Limitation of Horizontal Scale of Motions in Linear Geostrophic Adjustment Process

The governing equations of the large-scale motion for a barotropic fluid can be expressed in dimensionless form as follows:

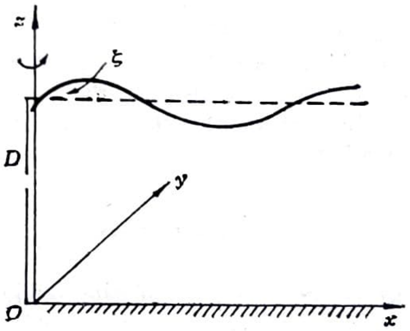

Fig. 1. Right-hand rectangular coordinates in which (x, y)-plane lies on the sea bottom with x eastward and y northward.

$$\varepsilon\frac{\partial \boldsymbol{V}_H}{\partial t}+\frac{F_r^{1/2}}{\alpha}\boldsymbol{V}_H\cdot\nabla_H\boldsymbol{V}_H=-\nabla_H\Phi-f\boldsymbol{k}\times\boldsymbol{V}_H, \tag{2.1}$$

$$\varepsilon\frac{\partial \Phi}{\partial t}+\frac{F_r^{1/2}}{\alpha}\boldsymbol{V}_H\cdot\nabla_H\Phi+\left(\frac{F_r^{1/2}}{\alpha}\Phi+\frac{1}{\alpha^2}\right)\nabla_H\cdot\boldsymbol{V}_H=0, \tag{2.2}$$

where $\varepsilon=\dfrac{1}{f_0T}$, $F_r=\dfrac{U^2}{gD}$, $L=L_0\alpha$, $L_0=\sqrt{gD}/f_0$; L denotes the characteristic horizontal length scale, L_0 the radius of Rossby deformation, D the mean depth of the fluid, T the characteristic scale for time, U the characteristic scale for horizontal velocity, f_0 the value of Coriolis parameter at the central latitude, α the controlling parameter of the horizontal scale, ∇ the two-dimensional gradient operator. The other nondimensional variables mentioned above have their usual meanings, as used in geophysical fluid dynamics.

Assumng $O(|-f\boldsymbol{k}\times\boldsymbol{V}_H-\nabla\Phi|)=O(1)$, it is evident from Eqs. (2.1) and (2.2) that the following condition must be satisfied if the process of geostrophic adjustment is linear,

$$O\left(\frac{F_r^{1/2}}{\alpha}\right) \leqslant 10^{-1}, \text{ i. e. } O(\alpha) \geqslant 100(F_r^{1/2}). \tag{2.3}$$

For the large-scale oceanic motions, we have $U \sim 10^{-1}$ m/s, $D \sim 10^3$ m and $g \sim 10$ m/s², and

$$O(F_r) = 10^{-3}, \text{ or its equivalence } O(\alpha) \geqslant O(10^{-2}).$$

This implies that the horizontal characteristic scale may differ in order of magnitude from the Rossby deformation radius in the linear geostrophic adjustment process of large-scale oceanic motions. Thus $L \ll L_0$ is permissive.

For the large-scale atmospheric motions, we also have $U \sim 10$ m/s, $D \sim 10^4$m and $g \sim 10$m/s², and this leads to

$$O(F_r) = 10^{-1}, \text{ or its equivalance } O(\alpha) \geqslant O(1).$$

Therefore, the order of magnitude of the characteristic horizontal scale is at least equal to the Rossby deformation radius. In other words, the condition $L < L_0$ with the same order of magnitude can be allowed.

Provided that the order of magnitude of $\frac{F_r^{1/2}}{\alpha}$ is much smaller than unity, Eqs. (2.1) and (2.2) can be reduced to

$$\varepsilon \frac{\partial \boldsymbol{V}_H}{\partial t} = -\nabla_H \Phi - f\boldsymbol{k} \times \boldsymbol{V}_H, \tag{2.4}$$

$$\varepsilon \frac{\partial \Phi}{\partial t} = -\frac{1}{\alpha^2} \nabla_H \cdot \boldsymbol{V}_H. \tag{2.5}$$

Now, let us consider the following three special cases.

(1) If $O(|-\nabla_H \Phi - f\boldsymbol{k} \times \boldsymbol{V}_H|) = O(1)$ and $O(|\nabla_H \cdot \boldsymbol{V}_H|) = O(1)$, we can easily get two different time scales from Eqs. (2.4)—(2.5), i. e.

$$O(T_1) = O\left(\frac{1}{f_0}\right) \text{ and } O(T_2) = O\left(\frac{\alpha^2}{f_0}\right).$$

To ensure the consistency of the time scale in the foregoing equations, the characteristic scale for time must be sole, that is, T_1 should be equal to T_2. Consequently,

$$O(\alpha) = 1, \text{ or equivalently, } O(L) = O(L_0).$$

Therefore, the horizontal scale of motions in the geostrophic adjustment process must be the same order of magnitude as that of the Rossby deformation radius when both the horizontal divergence and geostrophic departure possess the order unity. In addition, it is easily seen that the time scale for geostrophic adjustment is equal to f_0^{-1}.

(2) If $O(|-\nabla_H \Phi - f\boldsymbol{k} \times \boldsymbol{V}_H|) = O(1)$ and $O(|\nabla_H \cdot \boldsymbol{V}_H|) \neq O(1)$, then we get from Eqs. (2.4) and (2.5)

$$O(T) = O(f_0^{-1}); \ O(\alpha^2) = O(|\nabla_H \cdot \boldsymbol{V}_H|).$$

Thus, if the linear geostrophic adjustment process exists, the possible choice of L needs to be restricted to a certain extent. In other words, L must satisfy the constraint given above.

(3) If $O(|\nabla_H \cdot \boldsymbol{V}_H|) = O(1)$ and $O(|-f\boldsymbol{k} \times \boldsymbol{V}_H - \nabla_H \Phi|) \neq O(1)$, it leads to

$$O(T) = O(\alpha^2 f_0^{-1});O(\alpha^2|-f\boldsymbol{k} \times \boldsymbol{V}_H - \nabla_H \Phi|) = O(1).$$

It is obvious from the two restrictive conditions that both the ratio of the horizontal length scale to the Rossby deformation radius and the characteristic time for geostrophic adjustment are in inverse proportion to the geostrophic departure, that is, if the geostrophic departure is large, then the possibly selected value of L for the linear geostrophic adjustment process and the adjustment time will become small. The opposite conclusion holds true for a small geostrophic departure. This result is consistent with that obtained by Ye et al. (1965)[3].

III. Response of the Flow Field in the Oceanic Interior to a Steady Wind Field

Consider the large-scale motion in the oceanic interior for which the lateral eddy viscosity can be ignored. The effects of vertical eddy viscosity are introduced by using the Ekman pumping-induced vertical velocities of both lower and upper Ekman layers as the vertical boundary conditions for the interior flow. So, the governing equations and the corresponding boundary conditions in dimensional form can be simplified as:

$$\frac{\partial \boldsymbol{V}_H}{\partial t} = -f\boldsymbol{k} \times \boldsymbol{V}_H - \nabla_H \Phi, \tag{3.1}$$

$$\nabla_H \cdot \boldsymbol{V}_H + \frac{\partial W}{\partial z} = 0, \tag{3.2}$$

$$W|_{z=0} = \hat{K}\nabla^2\psi;\ W|_{z=D} = \frac{1}{g}\frac{\partial \Phi}{\partial t} + \frac{1}{f\rho}\boldsymbol{k} \times \text{curl}\,\boldsymbol{\tau}, \tag{3.3}$$

where $\hat{K} = \sqrt{\frac{\nu}{2f}}$; ν is the vertical eddy viscosity, ρ the sea water density, $\boldsymbol{\tau}$ the sea surface stress vector exerted by the wind, W the velocity in the vertical direction, ψ the stream function.

Substitution of $\boldsymbol{V}_H = \boldsymbol{k}\times\nabla_H\psi+\nabla_H\phi$ (ϕ: the velocity potential) into Eqs. (3.1) and (3.2), and invoking the boundary condition (3.3) yield:

$$\frac{\partial^3\psi}{\partial t^3} - C_0^2\nabla_H\frac{\partial\psi}{\partial t} + f^2\frac{\partial\psi}{\partial t} + \frac{g}{\rho}\boldsymbol{k} \times \text{curl}\,\boldsymbol{\tau} - gf\hat{K}\nabla_H^2\psi = 0. \tag{3.4}$$

Let ψ be divided into two parts, i. e. the steady geostrophic part ψ_g and the unsteady part $\tilde{\psi}$, which satisfy the following equations, respectively,

$$\frac{1}{\rho f}\boldsymbol{k} \times \text{curl}\,\boldsymbol{\tau} - \hat{K}\nabla_H^2\psi_g = 0, \tag{3.5}$$

$$\frac{\partial^3\tilde{\psi}}{\partial t^3} - C_0^2\nabla_H^2\frac{\partial\tilde{\psi}}{\partial t} + f^2\frac{\partial\tilde{\psi}}{\partial t} - gf\hat{K}\nabla_H^2\tilde{\psi} = 0. \tag{3.6}$$

It can be seen from Eqs. (3.5) and (3.6) that the steady geostrophic stream function satisfies the balance relation between the wind stress on the sea surface and the bottom friction, while the non-geostrophic perturbation is subject to the wave equation. Supposing that $\tilde{\psi}$ has the wave form of solution

$$\tilde{\psi} = Ae^{i(kx+\eta y-\sigma t)}. \tag{3.7}$$

In (3.6) (where σ, k, η denote the frequency and the wave numbers in x- and y- directions, respectively), we can immediately get the corresponding dispersion relation as follows:

$$\frac{1}{f^2 + C_0^2K^2}\sigma^3 - \sigma - \frac{gf\hat{K}K^2}{f^2 + C_0^2K^2}i = 0, \tag{3.8}$$

where $K^2 = k^2 + \eta^2$, $i = \sqrt{-1}$. Because $O\left(\frac{gf\hat{K}K^2}{f^2 + C_0^2K^2}\right) \leqslant O\left(\frac{f\hat{K}}{D}\right) \ll 1$, two distinct kinds of solution to (3.8) can easily be obtained:

$$\sigma_{1,2}^2 = f^2 + C_0^2K^2 + O\left(\frac{f\hat{K}}{D}\right), \tag{3.9}$$

$$\sigma_3 = -\frac{gf\hat{K}K^2}{f^2 + C_0^2K^2}i. \tag{3.10}$$

(3.9) represents the mixed waves due to the gravity-inertial effects which propagate towards two opposite directions, while (3.10) gives a kind of unpropagating and over-damping wave. The behavior of the latter is similar to the standing waves except that its amplitude decreases exponentially with time. Defining T as the time of wave decay, a simple estimate gives

$$T = \frac{\sqrt{2}\,(f^2 + C_0^2K^2)}{g\sqrt{f\nu}\,K^2}\pi. \tag{3.11}$$

Obviously, the time of decay is in inverse proportion to $\nu^{1/2}$ and directly proportional to D. In other words, the larger the eddy viscosity is (or the shallower the water depth is), the faster the wave will be dispersed.

It follows that in the response of the oceanic interior flow to a steady wind field the non-geostrophic perturbation energy is dispersed by the gravity-inertial waves and the unpropagating and over-damping waves caused by turbulent friction. The wind stress exerted on the sea surface and the bottom frictional force gradually approach the equilibrium, such that a quasi-geostrophic state of motion is ultimately established. Evidently, owing to the effect of turbulent diffusion, the geostrophic adjustment becomes speedy in comparison with that of inviscid fluid[5].

IV. Geostrophic Adjustment Process in the Baroclinic Ocean

1. *Internal Gravity-inertial Waves and Equilibrium Equations For Geostrophic Adjustment*

Consider the adiabatic, imcompressible and inviscid baroclinic ocean. Using the method of scale analysis, the linear dimensional equations governing the geostrophic adjustment process of the large-scale motion in the baroclinic ocean may be simplified to

$$\frac{\partial \boldsymbol{V}_H}{\partial t} = -f\boldsymbol{k} \times \boldsymbol{V}_H - \frac{1}{\rho_s} \nabla_H P, \tag{4.1}$$

$$\nabla_H \cdot \boldsymbol{V}_H + \frac{\partial W}{\partial z} = 0, \tag{4.2}$$

$$\frac{\partial \rho}{\partial t} = \frac{\rho}{g} N_s^2 W, \tag{4.3}$$

$$\frac{\partial}{\partial z}\left(\frac{P}{\rho_s}\right) = -\frac{\rho}{\rho_s} g, \tag{4.4}$$

where $\rho_s(z)$ denotes the density distribution of undisturbed ocean, $N_s^2 = -\frac{g}{\rho_s}\frac{\partial \rho_s}{\partial z}$ is the well-known Brunt-Väisälä frequency.The time of geostrophic adjustment for the baroclinic ocean can easily be estimated as

$$T \sim \left(\frac{L}{L_D}\right)^2 f_0^{-1}.$$

The length scale $L_D = DN_s/f_0$ is defined as the internal Rossby radius of deformation. Since $L_D < L_0$, the time required for the geostrophic adjustment is much longer in a baroclinic ocean than in a barotropic one. Moreover, all the foregoing results derived in Section II is held to be valid for the baroclinic ocean.

After some simple mathematical manipulation in Eqs. (4.1)—(4.4), the internal gravity-inertial wave and the equilibrium equations for geostrophic adjustment may be given as follows:

$$\left(\frac{\partial^2}{\partial t^2} + f^2\right)\frac{\partial^2 W}{\partial z^2} + N_s^2 \nabla_H^2 W = 0, \tag{4.6}$$

$$\frac{\partial^2 \bar{\Phi}}{\partial z^2} + \frac{N_s^2}{f^2}\left(\frac{\partial^2 \bar{\Phi}}{\partial x^2} + \frac{\partial^2 \bar{\Phi}}{\partial y^2}\right) = \Omega_0, \tag{4.7}$$

where $\Omega_0 = \frac{\partial^2}{\partial z^2}\left(\frac{P_0}{\rho_s f}\right) + \frac{N_s^2}{f^2}\nabla_H^2 \phi_0$; P_0 and ϕ_0 are the initial disturbances of pressure and stream function, respectively; $\bar{\Phi}$ is the steady geostrophic stream function.

It is obvious from Eqs. (4.6) and (4.7) that the non-geostrophic perturbation energy of large-scale motions in the baroclinic ocean is dispersed by the internal gravity-inertial waves and the whole adjustment process satisfies the conservation law of potential vorticity. Eqs. (4.6) and (4.7) are mostly the same in essence as those for the baroclinic atmosphere. However, the limited ocean bounds in the vertical and density stratification which are different from those in the atmosphere greatly affect the equilibrium state of geostrophic adjustment. Thus, many theoretical results about the geostrophic adjustment of baroclinic atmosphere is not adaptable to the baroclinic ocean. The details will be reported successively in the following.

2. *Geostrophic Adjustment Process in the Ocean of Finite Depth*

(1) *Energy dispersive characteristics of the disturbance*

Let us consider the baroclinic ocean of constant depth D, which extends infinitely in the horizontal extent. Both the bottom and the surface of the sea are assumed to be rigid surfaces, so

$$W|_{z=0}=0,\ W|_{z=D}=0$$

may serve as the boundary conditions of the problem under discussion. The initial conditions $W|_{t=0}=q_0(x,y,z)$ and $\left.\frac{\partial W}{\partial t}\right|_{t=0}=q_1(x,y,z)$ are assignable. The solution of Eq. (4.6) which satisfies the boundary and initial conditions given above is as follows:

$$W=\sum_l\left\{\frac{1}{\pi aD}\frac{\partial}{\partial t}\iint_{\Sigma_{at}^{x,y}}\frac{\cos\left(\frac{f}{a}\sqrt{a^2t^2-r^2}\right)}{\sqrt{a^2t^2-r^2}}\left[\int_0^D q_0(x+r\cos\theta,y\right.\right.$$

$$\left.+r\sin\theta,\zeta)\sin\frac{l\pi\zeta}{D}d\zeta\right]rdrd\theta+\frac{1}{\pi aD}\iint_{\Sigma_{at}^{x,y}}\frac{\cos\left(\frac{f}{a}\sqrt{a^2t^2-r^2}\right)}{\sqrt{a^2t^2-r^2}}$$

$$\left.\cdot\left[\int_0^D q_1(x+r\cos\theta,y+r\sin\theta,\zeta)\sin\frac{l\pi\zeta}{D}d\zeta\right]rdrd\theta\right\}\sin\frac{l\pi}{D}z, \tag{4.8}$$

where $a=\frac{N_sD}{l\pi}$, $0<1<+\infty$, $\Sigma_{at}^{x,y}$ denotes a circle of plane whose centre is at (x,y) and whose radius equals at.

The solution (4.8) means that the dispersive speed of non-geostrophic disturbance energy is closely related to the distribution of the initial unbalanced field as well as the magnitude of the parameters l, D, f and N_s. For simplicity, assuming the following initial disturbances,

$$q_0=\begin{cases}q_0(z)\text{ if } r\leqslant R;\\ 0\text{ if } r>R;\end{cases}\qquad q_1=\begin{cases}q_1(z)\text{ if } r\leqslant R,\\ 0\text{ if } r>R,\end{cases} \tag{4.9}$$

and taking $x=0$, $y=0$ and $at\geqslant R$, without loss of the generality of the solution (4.8), solution (4.8) can be reduced to

$$W=\sum_l\frac{2}{aD_l}\left\{\left(\int_0^D q_0(\zeta)\sin\frac{l\pi}{D}\zeta d\zeta\right)\frac{\partial}{\partial t}\left[\cos\left(\frac{f}{a}\sqrt{a^2t^2-\bar{r}^2}\right)\left(at-\sqrt{a^2t^2-R^2}\right)\right]\right.$$

$$\left.+\left(\int_0^D q_1(\zeta)\sin\frac{l\pi\zeta}{D}d\zeta\right)\cos\left(\frac{f}{a}\sqrt{a^2t^2-\bar{r}^2}\right)\left(at-\sqrt{a^2t^2-R^2}\right)\right\}\sin\frac{l\pi}{D}z, \tag{4.10}$$

where $0<\bar{r}\leqslant R$. This implies that $W\to 0$ as $at\gg R$. As a result, the motion approaches a geostrophic equilibrium state. The following results should be noted

1) If a is fixed, the speed of geostrophic adjustment is obviously in inverse proportion to R. In other words, the larger the horizontal scale R of the disturbance is, the slower the geostrophic adjustment will be. The opposite result is true for a smaller R. However, it is impossible to approach the geostrophic balance state as $R\to\infty$.

2) If R is a constant, the time required for geostrophic adjustment is in inverse proportion to a. Since

$$a=\frac{N_s D}{l\pi},$$

the larger l(i.e. wave nodal surface number) is (or the smaller either of D and N_s is), the shorter the time required for geostrophic adjustment will be. This result is coincident with B. Bolin's (1955)[1].

3) The vertical structure of the initial disturbance field is only related to the amplitude of waves, but is independent of the speed of the geostrophic adjustment.

(2) *Solution of the geostrophic equilibrium state equation*

Supposing that the density values of initial disturbance at both the sea surface and the bottom are equal to zero, the corresponding boundary conditions of Eq. (4.7) with $W|_{z=0}=0$ and $W|_{z=D}=0$ may be rewritten as

$$\left.\frac{\partial\bar{\Phi}}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial\bar{\Phi}}{\partial z}\right|_{z=D}=0. \tag{4.11}$$

The solution of Eq. (4.7) which satisfies the condition (4.11) is

$$\bar{\Phi}=-\sum_l \frac{f^2}{DN_s^2\pi}\left[\iint\limits_{-\infty}^{\infty}\left(\int_0^D \Omega_0\cos\frac{l\pi\zeta}{D}\,d\zeta\right)K_0\left(\frac{\sqrt{(x-\xi)^2+(y-\eta)^2}}{N_sD/l\pi f}\right)d\xi d\eta\right]$$
$$\cdot\cos\frac{l\pi}{D}z. \tag{4.12}$$

It follows from (4.12) that the solution of geostrophic equilibrium state is closely related to the horizontal and vertical distributions of the initial unbalanced field, the nodal surface number of waves in the vertical direction, the water depth, the latitude, the intensity of stratification and so on. No matter what the horizontal scale of disturbance is, the remarkable changes will take place between the flow and pressure fields because of the influences of the vertical boundary conditions. The phenomena appear no longer, in which the flow field remains approximately unchanged while the pressure field is adjusted to the flow field and *vice versa*. This is just one of the important distinctions between the geostrophic adjustment process in the ocean of finite depth and that in the baroclinic atmosphere.

We take a special example for further analysis as follows:

$$\psi_0=\psi_0(r)=-Ae^{-\frac{r^2}{2R^2}}\left(\frac{z}{D}\right)^2,\quad P_0=0, \tag{4.13}$$

where R is the distance from the centre of the vortex to the point of the maximum current velocity. $A>0(\mathrm{m^2/s})$. Substituting (4.13) into (4.12), the distributions of the geostrophic balanced flow and pressure are computed by numerical integration for several cases, such as $D=5000$, 2500 m and $N_s=10^{-3}\,s^{-1}$, $2\times10^{-3}\,s^{-1}$, where $A=2R^{1/2}$, R= 50 km and $f=10^{-4}\,s^{-1}$ (see Figs. 2—8). The computational results draw the following important conclusions: (i) The geostrophic equilibrium state is accomplished by the mutual adjustment between the flow and the pressure fields; (ii) major energy of geostrophic equilibrium state is concentrated on the solution for $l=1$, but all higher modes ($l\geq2$) make less contribution to the final state of equilibrium, and this result is in agreement with B. Bolin's[1]; (iii) it is easier to maintain the flow field unchanged in the strong stratification or in the deep water than in the weak stratification or in the shallow water, and the opposite conclusion is also true for the pressure field; (iv) apart from weakening of the current velocity in the upper layer, a vortex rotating in

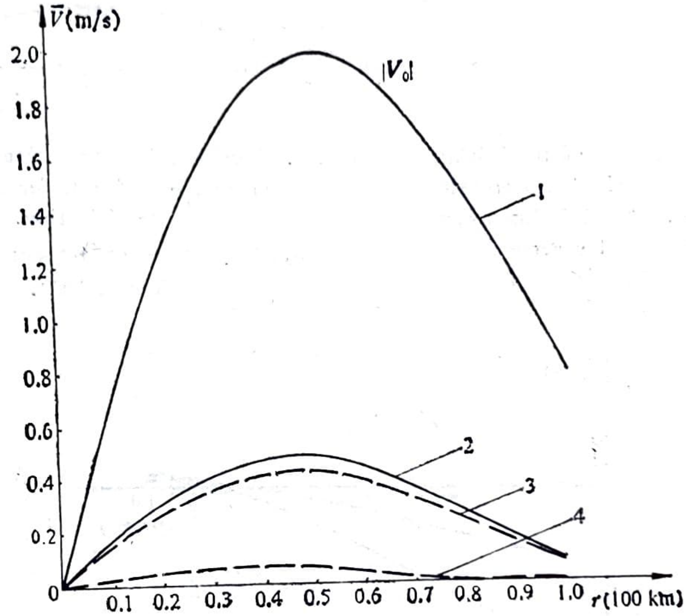

Fig. 2. Horizontal distributions of the initial and final geostrophic currents at the sea surface with parameters $R=50$ km, $L=100$ km, $f=10s^{-1}$, and $D=5000$ m. 1. The initial current; 2. sum of the currents after the adjustment obtained from $l=1$ to $l=8$; 3. current after the adjustment for $l=1$; 4. current after the adjustment for $l=2$.

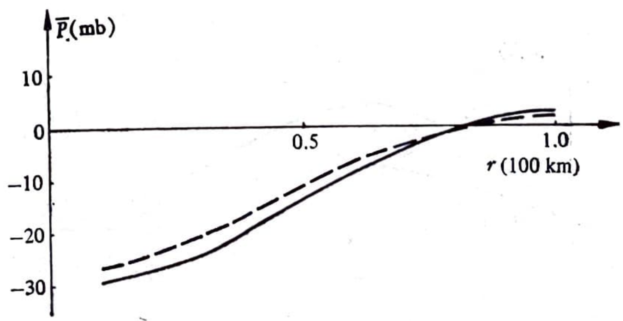

Fig. 3. Horizontal distributions of the pressure after the adjustment at the sea surface. Solid line represents the sum of the pressures after the adjustment obtained from $l=1$ to $l=8$, and the dashed line denotes the pressure after the adjustment for $l=1$.

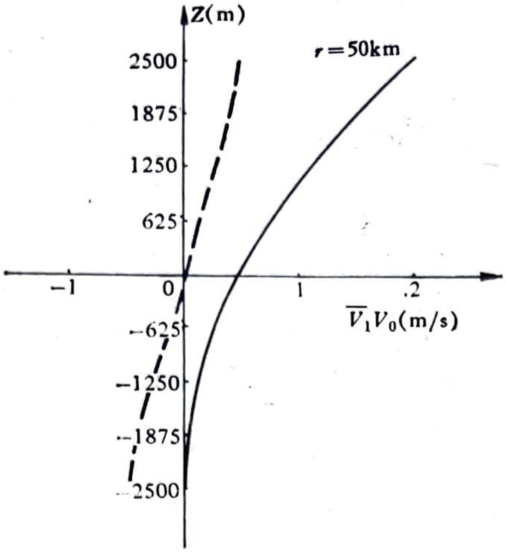

Fig. 4. Vertical profiles of the initial (solid line) and the final geostrophic (dashed line) currents at 50 km away from the centre of the vortex.

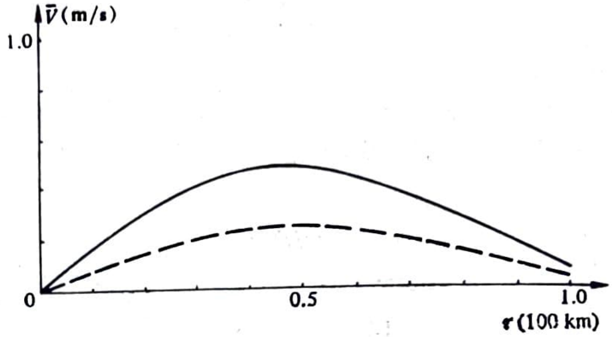

Fig. 5. Horizontal distributions of the current after the adjustment for different intensities of stratification.

Solid line: $N_s = 2 \times 10^{-3}s^{-1}$; dashed line: $N_s = 10^{-3}s^{-1}$.

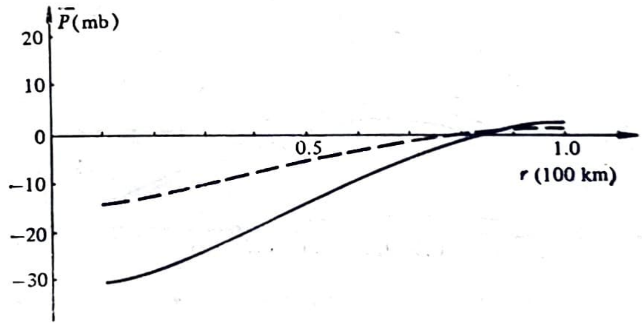

Fig. 6. Horizontal distributions of the pressure after the adjustment for different intensities of stratification.

Solid line: $N_s = 2 \times 10^{-3}s^{-1}$; dashed line: $N_s = 10^{-3}s^{-1}$.

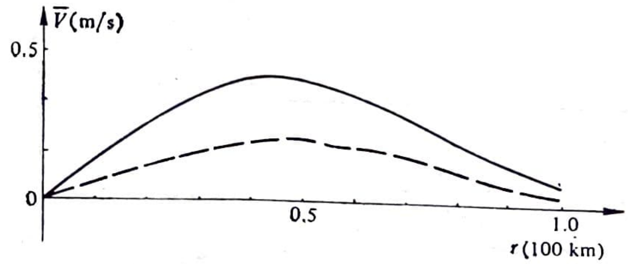

Fig. 7. Horizontal distributions of the current after the adjustment for different water depths.

Solid line: $D = 5000$ m; dashed line: $D = 2500$ m.

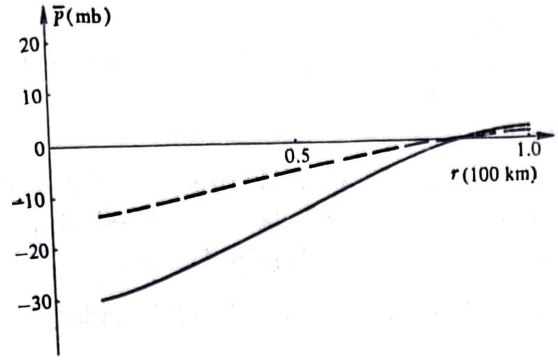

Fig. 8. Horizontal distributions of the pressure after the adjustment for different water depths.
Solid line: $D=5000$ m; dashed line: $D=2500$ m.

clockwise sense emerges in the lower layer, which is just opposite to that in the upper layer.

3. *Characteristics of Geostrophic Equilibrium State in a Two-layer Model Ocean*

For simplicity, we roughly divide the continuously stratified ocean into a simple two-layer model ocean on the basis of the observed profile of N_s in the real ocean, which satisfies the following conditions (Fig. 9).

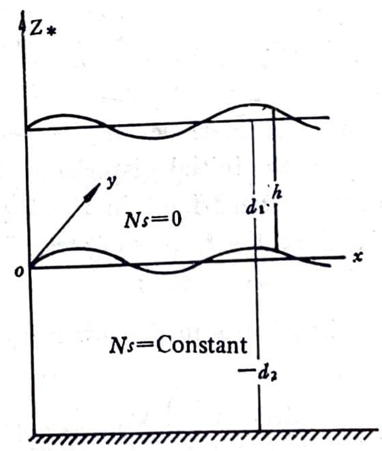

Fig. 9. Two-layer model ocean.

$$N_s = \begin{cases} 0, & \text{for the mixed layer, } 0 < z^* < d_1 \\ \text{constant,} & \text{for the deeper layer, } d_2 < z^* < 0. \end{cases}$$

In the upper layer, since $N_s=0$, the problem under consideration is the barotropic adjustment process in which the flow field is independent of z. Assuming the initial unbalanced field and the solution of equilibrium state can be expressed as the wave forms, the nondimensional solution of $\bar{\Phi}_{\text{I}}$ is thus given by

$$\bar{\Phi}_{\text{I}}(x, y) = \text{Re}\{Q_0 e^{i(kx+\eta y)}\}, \tag{4.14}$$

* Here asterisk is used to indicate the dimensional variable.

where $Q_0 = \dfrac{K^2\phi_{0I} + \left(\dfrac{L}{L_0}\right)\Phi_{0I}}{K^2 + \left(\dfrac{L}{L_0}\right)^2}$, $K = \sqrt{k^2 + \eta^2}$ is the nondimensional wave number; ϕ_{0I}, Φ_{0I} and $\bar{\Phi}_I$ denote the initial flow and pressure fields as well as the geostrophic balanced flow field in the upper layer, respectively. Subscript "I" refers to the variable in the upper layer and subscript "0" for the initial unbalanced field.

In the lower layer, assuming the nondimensional boundary conditions to be $\left.\dfrac{\partial \bar{\Phi}_{II}}{\partial z}\right|_{z=-1} = 0$ and $\bar{\Phi}_{II} = \bar{\Phi}_I(x, y)$ by the matching principle, and having

$$(\bar{\Phi}_{II}, P_{0II}, \phi_{0II}) = \mathrm{Re}(\hat{\phi}_{II}(z), \hat{P}_{0II}(z), \hat{\phi}_{0II}(z))e^{i(kx+\eta y)}, \tag{4.15}$$

(subcript "II" is used to indicate the variable in the lower layer), we can obtain the nondimensional solution of $\hat{\phi}_{II}(z)$ as follows:

$$\hat{\phi}_{II}(z) = \frac{1}{2\mathrm{ch}\, A}\left\{2Q_0\mathrm{ch}\, A(1+z) + \frac{1}{A}\left[\int_{-1}^{0} W(\xi)\mathrm{sh}\, A(z+1+\xi)d\xi \right.\right.$$
$$\left.\left. + \int_{\xi=0} W(\xi)\mathrm{sh}\, A(\xi - 1 - z)d\xi + \int_{\xi=-1} W(\xi)\mathrm{sh}\, A(1+\xi-z)d\xi\right]\right\}$$
$$+ \frac{1}{A}\int_{\xi=z} W(\xi)\mathrm{sh}\, A(z-\xi)d\xi, \tag{4.16}$$

where $W(\xi) = \dfrac{\partial^2 \hat{P}_{0II}(\xi)}{\partial \xi^2} - A^2\hat{\phi}_{0II}(\xi)$, $A = \dfrac{L_D K}{L}$, $L_D = \dfrac{N_s d_2}{f}$.

Solution (4.16) implies that the characteristics of the geostrophic equilibrium state of the lower layer motions depend on the initial disturbance in the upper layer, the vertical distribution of the initial unbalanced field in the lower layer and the horizontal scale of disturbance field as well as the factors of stratification and water depth. Several special cases are worthy of note as follows:

(1) Assuming the initial disturbance in the lower layer to be zero, hence

$$\hat{\phi}_{II}(z) = \frac{K^2\phi_{0I} + \left(\dfrac{L}{L_0}\right)^2\Phi_{0I}}{K^2 + \left(\dfrac{L}{L_0}\right)^2} \frac{\mathrm{ch}\, A(1+z)}{\mathrm{ch}\, A}. \tag{4.17}$$

For a small-scale disturbance

$$(\text{i. e. } L < L_0,\ L^2 \ll L_0^2),$$

A. If $\phi_{0I} \neq 0$, $\Phi_{0I} \ll \phi_{0I}$ (or $\Phi_{0I} \approx 0$), then

$$\hat{\phi}_{II}(z) = \hat{P}_{II}(z) = \phi_{0I}\frac{\mathrm{ch}\, A(1+z)}{\mathrm{ch}\, A}, \quad -1 < z < 0.$$

This means that the effects of the small-scale disturbance of the flow field in the upper layer on the geostrophic equilibrium state in the lower layer become weak with increasing depth of water.

B. If $\Phi_{0I} \neq 0$, $\phi_{0I} \ll \Phi_{0I}$ (or $\phi_{0I} \approx 0$), solution (4.17) can be approximately reduced to

$$\hat{\phi}_{II}(z) = \frac{L^2}{K^2 L_0^2} \Phi_{0I} \frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A}.$$

Since

$$\left(\frac{L}{L_0}\right)^2 \ll 1 \text{ and } \frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A} < 1,$$

we get

$$\hat{\phi}_{II}(z) \approx 0, \ \hat{P}_{II}(z) \approx 0.$$

Hence the small-scale disturbance of the pressure field in the upper layer exerts small influence on the geostrophic balance state in the lower layer. As a result, both the flow and the pressure fields in the lower layer remain approximately unchanged.

Similarly, for the large-scale disturbance of the pressure field, the effects of the initial disturbance field in the upper layer on the geostrophic equilibrium state in the lower layer become weak with increasing depth of the water, while for the large-scale disturbance of flow field, both the flow and the pressure patterns in the lower layer are easy to maintain.

(2) The initial disturbance in the upper layer is assumed to be zero (i.e. $Q_0=0$).

A. If the initial unbalanced state mainly acts as the disturbance of the pressure, i. e. $\hat{\phi}_{0II} \approx 0$, and that $\hat{P}_{0II}$ is an invariant constant or linear function of z, then

$$\hat{\phi}_{II}(z) = 0, \qquad \hat{P}_{II}(z) = 0.$$

Therefore, for the disturbance of pressure which is uniform or linear in the vertical, the pressure field is adjusted to the flow field, so that the motion approaches the geostrophic balance state. If

$$\hat{P}_{0II} = \frac{1}{2} z^2,$$

i. e. $\hat{P}_{0II}$ possesses the parabolic z-profile, then

$$\hat{\phi}_{II}(z) = \frac{1}{A^2}\left[\frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A} - 1\right], \ A = \frac{L_D K}{L}.$$

For a small-scale disturbance, $\hat{\phi}_{II}(z) \approx 0$ because

$$\frac{1}{A^2} = \left(\frac{L}{L_D K}\right)^2 \ll 1 \text{ and } \left|\frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A} - 1\right| \leqslant 1.$$

Consequently, for the small-scale disturbance of pressure, the pressure field is adjusted to the flow field, so that the motion approaches the geostrophic balance state. By analogy, for a large-scale disturbance, since $\frac{1}{A^2} \gg 1$ and $\frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A} - 1 < 0$, the geostrophic balance state is accomplished by the mutual adjustment between the flow and the pressure fields.

B. Provided that the initial unbalanced state mainly acts as the disturbance of flow field (i.e. $\hat{P}_{0II} \approx 0$) and $\phi_{0II}(z) = 1$, i.e. the disturbance of flow which is uniform in the vertical, we obtain from (4.16)

$$\hat{\phi}_{II}(z) = 1 - \frac{\operatorname{ch} A(1+z)}{\operatorname{ch} A}.$$

It is concluded that the geostrophic equilibrium state is established by the adaptation of the flow field to the pressure field for the large-scale disturbance (i.e. $L_D \ll L, A \ll 1$). The opposite conclusion is true for the small-scale disturbance (i.e. $L \ll L_D, A \gg 1$). In addition, the more generalized cases of the problem are able to be discussed by means of numerical computation and will not be mentioned here.

V. Conservation of Potential Vorticity in Nonlinear Process of Geostrophic Adjustment

By means of the small parameter expansion in Rossby number R_0, the governing adjustment equations of the first-order approximation can be readily derived as

$$\frac{\partial \boldsymbol{V}_1}{\partial t} + f\boldsymbol{k} \times \boldsymbol{V}_1 = -\frac{1}{\rho_s} \nabla_H P_1 + \boldsymbol{A}, \tag{5.1}$$

$$\frac{\partial}{\partial z}\left(\frac{P_1}{\rho_1}\right) = -\frac{\rho_1}{\rho_s} g, \tag{5.2}$$

$$\nabla_H \cdot \boldsymbol{V}_1 + \frac{\partial W_1}{\partial z} = 0, \tag{5.3}$$

$$\frac{\partial \rho_1}{\partial t} - \frac{\rho_s}{g} N_s^2 W_1 = A_3, \tag{5.4}$$

where $A_3 = -\left(\boldsymbol{V}_0 \cdot \nabla_H \rho_0 + \rho_s W_0 \frac{\partial}{\partial z}\left(\frac{\rho_0}{\rho_s}\right)\right)$ and

$$\boldsymbol{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} = -\left(\boldsymbol{V}_0 \cdot \nabla_H \boldsymbol{V}_0 + W_0 \frac{\partial \boldsymbol{V}_0}{\partial z}\right)$$

represent the density advection and velocity advection, respectively. Subscript "1" denotes the first-order variable, and "0" the zeroth-order variable.

Supposing N_s to be a constant and combining Eqs. (5.1)—(5.4) with a little mathematical manipulation, it leads to

$$\frac{\partial}{\partial t}\left[\nabla_H^2 \phi_1 + \frac{f}{N_s^2} \frac{\partial}{\partial z^2}\left(\frac{P_1}{\rho_s}\right)\right] = \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} - \frac{fg}{N_s^2} \frac{\partial}{\partial z}\left(\frac{A_3}{\rho_s}\right), \tag{5.5}$$

where

$$\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} = -\left[(\nabla_H^2 \phi_0)(\nabla_H^2 \phi_0) + (\boldsymbol{V}_0 \cdot \nabla_H)\nabla_H^2 \phi_0 + W_0 \frac{\partial}{\partial z}(\nabla_H^2 \phi_0)\right.$$
$$\left. + \frac{\partial W_0}{\partial z}\frac{\partial v_0}{\partial z} - \frac{\partial W_0}{\partial y}\frac{\partial u_0}{\partial z}\right]$$

and

$$\frac{\partial}{\partial z}\left(\frac{A_3}{\rho_s}\right) = -\frac{\partial}{\partial z}\left[\nabla_H \cdot \left(\frac{\rho_0}{\rho_s}\right)\boldsymbol{V}_0 + \frac{\partial}{\partial z}\left(\frac{W_0\rho_0}{\rho_s}\right)\right].$$

It is shown from Eq. (5.5) that the first-order potential vorticity is no longer conserved because of the nonlinear terms comprising the zeroth-order variables. The local rate of change in potential vorticity depends on the magnitudes of the zeroth-order nonlinear terms such as the vorticity advection, the vertical transport of the horizontal vorticity, the mass flux and nonlinear interaction between the vorticity and the divergence fields. However, as was pointed out earlier[3], in the geostrophic adjustment process, the potential flow is much stronger than the rotational flow, and W is very important for the baroclinic fluid. Therefore, we can further simplify the nonlinear terms. Ignoring the vorticity advection and the vertical transport of the horizontal vorticity in the flow field as well as the horizontal transport of density in the mass field, that is, taking account of nothing but the nonlinear interaction between vorticity and divergence fields and the vertical transport of mass field, we have

$$\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} = \frac{1}{2f}\frac{\partial}{\partial t}[\nabla_H^2\psi_0]^2;$$

$$\frac{\partial}{\partial z}\left(\frac{A_3}{\rho_s}\right) = -\frac{1}{2N_s^2 g}\frac{\partial}{\partial t}\left[\frac{\partial^2}{\partial z^2}\left(\frac{\partial}{\partial z}\left(\frac{P_0}{\rho_s}\right)\right)^2\right]. \tag{5.6}$$

Substitution of (5.6) into (5.5) yields

$$\frac{\partial}{\partial t}\left[\frac{\partial^2}{\partial z^2}\left(\frac{P_1}{\rho_s}\right) + \frac{N_s^2}{f}\nabla_H^2\psi_1 - \frac{N_s^2}{2f^2}(\nabla_H^2\psi_0)^2 - \frac{1}{2N_s^2}\frac{\partial^2}{\partial z^2}\left(\frac{\partial}{\partial z}\left(\frac{P_0}{\rho_s}\right)\right)^2\right] = 0, \tag{5.7}$$

or equivalently

$$\frac{\partial^2\bar{\Phi}_1}{\partial z^2} + \frac{N_s^2}{f^2}\nabla_H^2\bar{\Phi}_1 = \Omega_1, \tag{5.8}$$

where

$$\Omega_1 = \frac{N_s^2}{2f^3}(\nabla_H^2\bar{\Phi}_0)^2 + \frac{f}{2N_s^2}\frac{\partial^2}{\partial z^2}\left(\frac{\partial\bar{\Phi}_0}{\partial z}\right)^2 - \frac{N_s^2}{2f^3}(\nabla_H^2\psi_0')^2 - \frac{1}{2N_s^2 f}\frac{\partial^2}{\partial z^2}\left(\frac{\partial}{\partial z}\left(\frac{P_0'}{\rho_s}\right)\right)^2.$$

$\bar{\Phi}_0$ indicates the stream function of the zeroth-order equilibrium state; ψ_0' and P_0' are the initial disturbance stream function and pressure of the zeroth-order variables, respectively; $\bar{\Phi}_1$ is the stream function of the first-order in the equilibrium state (Note that the initial disturbances of the first-order (P_1, ψ_1) are assumed to be zero).

It follows that the potential vorticity for the first-order model is conserved in the nonlinear geostrophic adjustment process only when the nonlinear effects of the potential flow and vertical flow are introduced.

VI. Concluding Remarks

From the foregoing discussion, the main results may be summarized as follows.

(1) In the linear geostrophic adjustment process of large-scale motion, the horizontal scale of motion is restricted to a certain extent, and such qualifications are

stronger in the atmosphere than in the ocean.

(2) In the response of the flow in the oceanic interior to a steady wind field, the energy of non-geostrophic disturbance is dispersed by the gravity-inertial waves and an unpropagating and over-damping wave caused by turbulent friction. Ultimately, the surface wind stress and bottom frictional force approach the balance state so that a quasi-geostrophic motion is established. In addition, the geostrophic adjustment becomes faster in consideration of the effect of turbulent diffusion.

(3) The speed of the geostrophic adjustment is in inverse proportion to the horizontal length scale but directly proportional to the intensity of stratification and depth of water. The vertical structure of disturbance is only related to the amplitude of waves, but is independent of the geostrophic adjustment speed.

(4) The limited bounds of ocean in the vertical direction directly affect the features of the solution of geostrophic equilibrium state. In the ocean of finite depth, the geostrophic equilibrium state is reached by the mutual adjustment between the flow and the pressure fields. Moreover, it is easier to maintain the flow field unchanged in the strong stratification or in the deep water than in the weak stratification or in the shallow water. The opposite conclusion is true for the pressure field.

(5) In the two-layer model ocean, the effects of the small-scale disturbance of the flow field in the upper layer on the geostrophic equilibrium state in the lower layer become weaker with increasing depth of water, while the small-scale disturbance of the pressure field in the upper layer has small influence on the geostrophic balance state in the lower layer. The opposite results are true for the large-scale disturbance. In addition, the characteristics of the geostrophic equilibrium state of the lower layer motions are closely related to the vertical structure of the lower layer disturbance. When the initial unbalanced state mainly acts as the disturbance of pressure whose distribution is uniform or linear in the vertical direction, the pressure field is adjusted to the flow field so that the motion approaches the geostrophic equilibrium state. When the initial unbalanced state mostly acts as the disturbance of flow which is uniform in the vertical, for the large-scale disturbance the geostrophic equilibrium state is established by the adjustment of the flow field to the pressure field. The opposite result is true for the small-scale one.

(6) The potential vorticity of the first-order model is conserved when merely the nonlinear potential flow and vertical transport of mass field are taken into consideration for the nonlinear terms.

Finally, it should be pointed out that all the discussions given above about the geostrophic adjustment process of the baroclinic ocean are only valid for the inviscid fluid in which the turbulent frictional force and the source of heat can safely be neglected. However, there exist the adaptation processes of motion to the exterior source of heat and the interior turbulent frictional field in the real ocean. In the shallow sea, especially near the coastal zone, the nonlinear process and the effect of the lateral friction become quite remarkable. Thus, it is more significant to deal with the response of the ocean current to the external source of heat and the interior turbulent frictional field as well as the nonlinear adjustment process with the effect of lateral friction. Certainly, all of these investigations will help us go deeper insight into the mechanism of the interaction between the ocean and atmosphere as well as the dynamic processes in the shallow sea. These problems remain to be further studied.

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